Math 45 SSM 2/e 3.5 Point-Slope Form of Equation of a Line

Objectives:
1) Find the equation of a line when given a point (x_1, y_1) and the slope m. using the point-slope formula.
2) Find the equation of a line when given two points (x_1, y_1) and (x_2, y_2) using the slope formula and the point-slope formula.
3) Build linear models using the point-slope formula. (i.e. word problems).

As a way to help remember the point-slope formula:

slope formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

replace x_2 by x and y_2 by x (any point on the line)

$$m = \frac{y - y_1}{x - x_1}$$

multiply both sides of equation by $(x - x_1)$

$$m \cdot (x - x_1) = \frac{(y - y_1)}{(x - x_1)} \cdot (x - x_1)$$

$$m \cdot (x - x_1) = y - y_1$$

OR $\boxed{y - y_1 = m(x - x_1)}$ POINT-SLOPE FORMULA

* Must memorize!

Examples

① Find the equation of a line whose slope is 3 and passes through $(-1, 2)$. Write in slope-intercept form. Graph the line.

step 1: Plug into point-slope formula $(x_1, y_1) = (-1, 2)$, $m = 3$.

$y - y_1 = m(x - x_1)$

$y - 2 = 3(x + 1)$

step 2: distribute and rearrange to $y = mx + b$.

$y - 2 = 3x + 3$

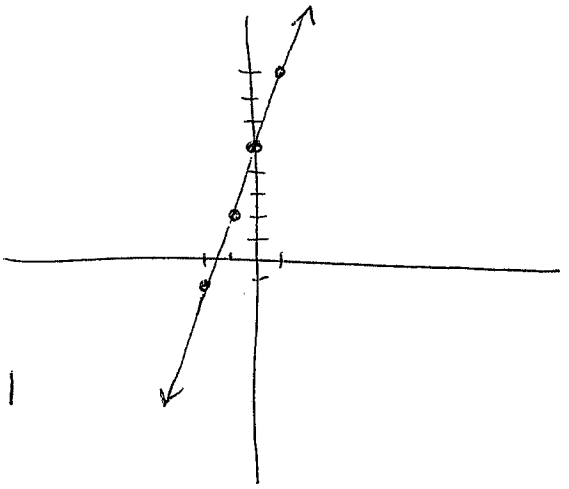
$+2 \qquad +2$

$\boxed{y = 3x + 5}$

cont →

① cont.

Graph $y = 3x + 5$



plot y-int (0,5)

go up 3, right 1

$m = \frac{3}{1}$

and down 3, left 1

$m = \frac{-3}{-1}$

② Write the equation of a line with slope $-\frac{2}{5}$, contains $(-4, 3)$.

> **Note:** "passing through" and "contains" both mean that the point is on the line.

step 1: point-slope formula

$$y - y_1 = m(x - x_1)$$

$$\begin{cases} m = -\frac{2}{5} \\ x_1 = -4, \quad y_1 = 3 \end{cases}$$

$$y - 3 = -\frac{2}{5}(x - (-4))$$

step 2: distribute and rearrange to $y = mx + b$, combine

$$y - 3 = -\frac{2}{5}x - \frac{2}{5}(4)$$

$$y - 3 = -\frac{2}{5}x - \frac{8}{5}$$

$$+3 \qquad +3$$

$$y = -\frac{2}{5}x - \frac{8}{5} + \frac{3}{1}\cdot\frac{5}{5} \qquad \text{common denom}$$

$$y = -\frac{2}{5}x - \frac{8}{5} + \frac{15}{5}$$

$$\boxed{y = -\frac{2}{5}x + \frac{7}{5}}$$

③ Write the equation of a line through $(1, 3)$ and $(4, 9)$. (x_1, y_1) (x_2, y_2)

in slope intercept form.

step 1: Find slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 3}{4 - 1} = \frac{6}{3} = 2.$$

③ cont

step 2: Plug into point-slope formula.

Note: You get the same equation if you use (1,3) as if you use (4,9). See options 1 & 2.

option 1: Use $(x_1, y_1) = (1,3)$.

$y - y_1 = m(x - x_1)$

$y - 3 = 2(x - 1)$

$y - 3 = 2x - 2$

$+3 \qquad +3$

$\boxed{y = 2x + 1}$ ← using (1,3)

option 2: Use $(x_1, y_1) = (4, 9)$

$y - 9 = 2(x - 4)$

$y - 9 = 2x - 8$

$+9 \qquad +9$

$\boxed{y = 2x - 1}$ ← using (4,9)

④ Write the equation of a line through $(-2, 4)$ and $(2, -2)$ in slope intercept form.
(x_1, y_1) (x_2, y_2)

step 1: find slope $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-2 - 4}{2 - (-2)}$

$= \frac{-6}{4}$

$m = \frac{-3}{2}$

step 2: point-slope formula

$y - y_1 = m(x - x_1)$

option 1: $(-2, 4)$

$y - 4 = -\frac{3}{2}(x + 2)$

$y = -\frac{3}{2}x - \frac{3}{2} \cdot 2 + 4$

$y = -\frac{3}{2}x - 3 + 4$

$\boxed{y = -\frac{3}{2}x + 1}$

option 2 $(2, -2)$

$y + 2 = -\frac{3}{2}(x - 2)$

$y = -\frac{3}{2}x + \frac{3}{2} \cdot 2 - 2$

$y = -\frac{3}{2}x + 3 - 2$

$\boxed{y = -\frac{3}{2}x + 1}$

(5) Example

The % of total income [y] that an individual spends on healthcare [increases] [linearly] with age [x]

← y values are %
← x values are age
← "increases" means slope is positive
← "linearly" means linear equation, or line.

A 35-year-old spends 4% on healthcare.
A 65-year-old spends 11.2% on healthcare.

→ when x=35, y=4 point (35, 4)
→ when x=65, y=11.2 point (65, 11.2)

a) Find slope
b) Find equation of line in slope-int form.
c) Use equation to predict % that a 50-year-old spends.

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a) slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

plug in $(35, 4)$ (x_1, y_1) and $(65, 11.2)$ (x_2, y_2)

$$m = \frac{4 - 11.2}{65 - 35} = \frac{7.2}{30} = \boxed{.24 = m}$$

b) point-slope form of eqn of a line: $y - y_1 = m(x - x_1)$.
plug in $m = .24$, $x_1 = 35$, $y_1 = 4$: $y - 4 = .24(x - 35)$
dist : $y - 4 = .24x - 8.4$
add 4 : $\boxed{y = .24x - 4.4}$

c) Predict 50-year-old. age=x so x=50.
Plug in x=50, solve for y. $y = .24(50) - 4.4$
$y = \boxed{7.6\% \text{ of income spent on healthcare}}$

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Extra Examples

⑥ Write equation of a line through $(0.7, 0.8)$ and $(0.2, 0.4)$ in slope-intercept form.

step 1: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0.8 - 0.4}{0.7 - 0.2} = \frac{0.4}{0.5} = \frac{4}{5}$

step 2: $y - y_1 = m(x - x_1)$

$y - 0.4 = \frac{4}{5}(x - 0.2)$

Option 1: change decimals to fractions $\begin{cases} .4 = \frac{4}{10} \\ .2 = \frac{2}{10} \end{cases}$

$y - \frac{4}{10} = \frac{4}{5}(x - \frac{2}{10})$

reduce fractions

$y - \frac{2}{5} = \frac{4}{5}(x - \frac{1}{5})$

dist and rearrange

$y = \frac{4}{5}x - \frac{4}{5}\cdot\frac{1}{5} + \frac{2}{5}$

$y = \frac{4}{5}x - \frac{4}{25} + \frac{2}{5}\cdot\frac{5}{5}$

$y = \frac{4}{5}x - \frac{4}{25} + \frac{10}{25}$

$\boxed{y = \frac{4}{5}x + \frac{6}{25}}$

Option 2: change fractions to decimals $\left\{ \frac{4}{5} = 4 \div 5 = .8 \right.$

$y - 0.4 = .8(x - 0.2)$

$y = .8x - .8(0.2) + 0.4$

$y = .8x - .16 + 0.4$

$\boxed{y = .8x + .24}$

check: $4 \div 5 = \frac{4}{5} = .8$ ✓

$6 \div 25 = \frac{6}{25} = .24$ ✓

⑦ Write the equation of a line through $\left(\frac{1}{3}, \frac{12}{5}\right)$ and $\left(\frac{4}{3}, \frac{2}{5}\right)$. in slope-intercept form.

step 1: $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{\left(\frac{12}{5} - \frac{2}{5}\right)}{\left(\frac{1}{3} - \frac{4}{3}\right)}$

$= \frac{\left(\frac{10}{5}\right)}{\left(\frac{-3}{3}\right)}$ ← subtract numerators ← subtract denominators

$= \frac{2}{-1}$ reduce results.

$= -2$ reduce

step 2: point-slope formula

Option 1: use $\left(\frac{4}{3}, \frac{2}{5}\right)$

$y - \frac{2}{5} = -2\left(x - \frac{4}{3}\right)$

$y = -2x + 2 \cdot \frac{4}{3} + \frac{2}{5}$

$y = -2x + \frac{8}{3} \cdot \frac{5}{5} + \frac{2}{5} \cdot \frac{3}{3}$

$y = -2x + \frac{40}{15} + \frac{6}{15}$

$\boxed{y = -2x + \frac{46}{15}}$

Option 2: use $\left(\frac{1}{3}, \frac{12}{5}\right)$

$y - \frac{12}{5} = -2\left(x - \frac{1}{3}\right)$

$y = -2x + 2 \cdot \frac{1}{3} + \frac{12}{5}$

$y = -2x + \frac{2}{3} \cdot \frac{5}{5} + \frac{12}{5} \cdot \frac{3}{3}$

$y = -2x + \frac{10}{15} + \frac{36}{15}$

$\boxed{y = -2x + \frac{46}{15}}$

Extra practice

Write the equation in slope-int form satisfying:

(1) through $(-1, 2)$, slope -2

(2) through $(0, 4)$ and $(-2, 0)$

~~(3) vertical through $(-3, 10)$~~

~~(4) through $(3, 12)$ with undefined slope~~

~~(5) through $(-2, 4)$ and $(2, -2)$~~

~~(6) horizontal through $\left(\frac{1}{2}, \frac{7}{4}\right)$~~

~~(7) through $(-6, 5)$ and $(7, 5)$~~

~~(8) through $(5, -7)$ slope 0.~~

~~(9) through $(0.7, 0.8)$ and $(0.2, 0.4)$~~ } (8½) through $(1.3, 0.4)$ and $(0.9, 0.3)$

~~(10) through $\left(\frac{1}{3}, \frac{12}{5}\right)$ and $\left(\frac{4}{3}, \frac{2}{5}\right)$~~

(11) through $(-3, 0)$ with slope $\frac{4}{5}$.

(12) In 1975, there were 39,161 fatal crashes in the U.S.
2002 38,309

Let y = # fatalities.
x = # years after 1975.

a) fill in ordered pairs $(\_\_, 39161)$ $(\_\_, 38309)$
b) plot the points & graph the line.
c) find the linear equation
d) use eqn to predict fatalities in 1985.
e) interpret slope